Transparency and Volatility: A Study of Electronic Security Trading Platform with Algorithmic Trading

Li, Xihao*

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Abstract

Under the background of the electronic security trading platform Xetra operated by Frankfurt Stock Exchange, we consider the Xetra auction market system (XAMS) from 'bottom-up', which the interaction among heterogeneous traders and Xetra auction market mechanism generates non-equilibrium price dynamics. We construct the corresponding agent-based model of XAMS and conduct market experiments with the Monte Carlo computer simulation. Then we investigate the role of the price setter who assumes its trading behavior can manipulate the market price. The main finding is that the introduction of the price setter in the setting of XAMS improves market efficiency while does not significantly influence price volatility of the market.

Keywords: market microstructure, transparency, volatility, algorithmic trading, electronic security trading platform, auction market.

JEL Classification: B4, C6, C9, D4, D5, D6, G1.

*Department of Economics and Social Sciences (DiSES), Università Politecnica delle Marche, Piazzale Martelli 8, 60121 Ancona, Italy.
Email: xihao.li@gmail.com.

1 Introduction

The advance in Information Technology has constantly injected faster and cheaper computational power in every corner of our society, financial industry is with no exception. As a consequence, new pattern of trading has risen in financial industry. On one hand, we have witnessed the prevalence of electronic security trading platforms in global markets. On the other hand, it becomes fashionable among players in electronic trading platforms to promote algorithmic trading - the use of computer algorithm to automatically execute order flows to trade in electronic platforms. The driving force of this fashionable trend in algorithmic trading is on its profitability. It becomes inevitable for the prevailing on this pattern of trading.

As the other side of the same coin, electronic trading platforms provide extensive market transparency, when combining with extensive employment of algorithmic trading, have brought us unprecedented speed of market crashes, for example, in 2010 US Flash Crash, US financial markets recorded within 30 minutes its biggest one-day point decline on the intraday basis in history, see SEC report (CFTC/SEC, 2010). This brings a new challenge to regulation in this type of digital markets, specially concerned with the market volatility under "the race to zero", e.g. see (Haldane, 2011). It leads to an open issue on studying in the field of economic research the relationship between market transparency and market volatility under the new environment of electronic trading platforms with prevailing algorithmic trading.

The relationship between market transparency and volatility has been pervasively investigated in the economic literature of market microstructure. For example, theoretical research in (Madhavan, 1996) develops a model of imperfect information game between strategic traders in auction markets. It shows transparency can increase price volatility and lower liquidity due to the increasing effects of asymmetric information between traders. Contrary result has been obtained in (Pagano & Röell, 1996) which considers order book transparency, and compares the effect of market transparency in auction markets and dealer markets with informed traders and uninformed traders. It shows these market mechanisms are attached with different levels of transparency, and higher degree of transparency generally lowers trading costs for uninformed traders while it does not necessarily increase price variability as the implicit bid-ask spread gets tighter.

Empirically, the relationship between transparency and volatility has been examined with market data among global stock exchanges. For example, (Madhavan *et al.*, 2005) shows that an increase in market transparency on Toronto Stock Exchange in 1990 leads to an increase in price volatility. Contrary result proposes in (Boehmer *et al.*, 2005), which investigates NYSE market data. It shows an increase in pre-trade transparency attracts traders to manage limit-order exposure by submitting smaller orders with less price impacts, thus results in an improvement in certain dimensions of market quality. (Sakawa & Ubukata, 2012) investigates the market data in Tokyo Stock Exchange. It shows an increase in market transparency insignificantly improves market volatility.

Other research results has been investigated in the survey of (Madhavan, 2000), (Stoll, 2003), and (Biais *et al.*, 2005), among others. The research in the strand of market microstructure has a mixed conclusion that the relationship between market transparency and market volatility depends on specific forms of market mechanism such as (opening/closing) auctions or continuous trading. Market transparency tends to be linked with information asymmetry among different groups of traders, which has impact on market volatility.

Another strand of research one may consult is on the role of algorithmic trading in electronic trading platforms. In this strand, algorithmic trading is also coined with the name of high frequency trading (HFT). Empirically, (Gerig, 2012) studies NASDAQ data. It shows price synchronization generated by HFT leads to market instability during times of stress. Contrary finding is shown in (Hendershott & Riordan, 2011b), (Brogaard, 2011), and (Hasbrouck & Saar, 2013). These authors share the finding that HFT does not increase intraday volatility in NASDAQ market. (Hendershott & Riordan, 2011a) and (Groth, 2011) analyze the market data in Deutsche Boerse. Both find no positive correlation between HFT and market volatility in German markets. (Kirilenko *et al.*, 2011) conducts an empirical case study on the 2010 US Flash Crash. It concludes that HFTs did not trigger the Flash Crash, but they amplified market volatility when responding to the unusual selling pressure.

Theoretically, Martinez & Rosu (2011) develops an equilibrium model by following the classical model in Kyle (1985). It shows HFTs does not destabilize markets, although HFTs generate a large percentage of trading volume and price volatility. Jarrow & Protter (2012) models HFT to trade quickly to exploit the mispricing or the market signal. It shows the introduction of HFT both increases market volatility and generates abnormal opportunities for HFT at the expense of slow traders. It shows a mixed finding that algorithmic trading generates low latency which lowers market volatility, while large trading volumes has a negative impact on price volatility. Although this strand of research comes to no consensus on whether market transparency with participation of HFT and algorithmic trading leads to market volatility or not, it indicates that algorithmic trading promotes a new type of information asymmetry, i.e. HFT has the speed advantage for profit exploitation by realizing its order specification in a faster pace than conventional traders with low trading frequency, see Foucault *et al.* (2013). On the other hand, it has difficulty to develop a formal model for strategies that HFT adopts to utilize the speed advantage for profit, for the reason that these strategies are at the core of the profit generator for trading firms, which remains black-box for outsiders. Another factor that contributes to the difficulty is that strategies on HFT are contingent to specific form of market mechanism that is generically complex computational process.

In view of this difficulty, we apply a pragmatic perspective to conduct our research. First, we consider as background a specific form of market mechanism, (opening/closing) auction conducted in Xetra, the pan-European electronic trading system operated by Deutsche Boerse, see Deutsche Börse Group (2013). Xetra auction belongs to the type of multi-unit double auction. It mainly consists of the call phase and the price determination phase. During the call phase, traders submit to the system order specifications, e.g. market orders and limit orders. The system collects these orders into a central order book without giving rise to transactions. The price determination phase follows when the call phase stops randomly after a fixed time span. Given the central order book, the system applies Xetra auction market mechanism (XAMM) determines the trading price and the trading volume in the price determination phase by applying Xetra auction trading rules specified in Deutsche Börse Group (2013). Under the background of Xetra auction market, we assume pre-trade transparency as the market transparency, i.e. during the call phase, traders obtain real-time information about the central order book. This assumption is mainly for simplifying our modeling work, but considering the current Xetra auction provides the real-time information for the first five best prices listed on both the buy side and the sell side of the central order book during the call phase, this assumption is by no means unrealistic.

We then develop the representation of HFT with speed advantage by the role of price setter that is assumed to obtain first priority for its order execution in Xetra auction market. We further assume, besides pre-trade transparency, the price setter has full knowledge on the market microstructure of Xetra auction, which implies that, by submitting calculated order specification, the price setter has the capability of intentional price manipulation against the central order book. When combining with the assumption that the price setter's order specification is to be executed with first priority, We grant the price setter the trading strategy of utilizing the market transparency given by Xetra auction market and its speed advantage for profit exploitation by means of price manipulation.

To investigate the relationship between market transparency and market volatility in Xetra auction market, we develop a market experiment to compare the market dynamics generated by Xetra auction market system (XAMS) with the participation of the price setter with that generated by a benchmark of Xetra auction market system without the participation of the price setter. This market experiment is involved with modeling XAMM to determine the market price and the trading volume as well as modeling the price setter's trading strategy, which are generically computational process. In this regard, we develop the market experiment by computer simulation with the methodology of agent-based modeling.

Computer simulation with the methodology of agent-based modeling in studying trading behavior and market mechanism is not new, e.g. see LeBaron (2006) for an early review of applying agent-based modeling in financial market. A more recent study is shown in Gsell (2008) that simulates with two types of traders in continuous double auction market: stylized traders (combination of informed trader, momentum trader, noise trader) and algorithmic traders. This strand of study tends to use simplified model of market mechanism such as continuous double auction or single-unit double auction. In our work, we aim at including the actual auction market mechanism, XAMM. The inclusion of XAMM has influences in two aspects. First, XAMM determines the trading price and trading volume in market dynamics. Secondly, we have to provide explicit model on how the trader utilize the knowledge of XAMM for its trading strategy. We show how to handle these two aspects by developing the agent-based model of XAMS in section 2. Then with the implementation of the agent-based model by employing the computer programming language Groovy/Java, we conduct in section 3 the computational market experiment to simulate XAMS and preform statistical analysis on simulation results to compare the market dynamics for XAMS and the benchmark system. 4 concludes with a brief discussion on implications and possible extensions of our work.

2 Agent-based Model of Xetra Auction Market System

We follow the methodology for standardized agent-based modeling proposed in Li (2014). Consider an economy with one risky asset market and one risk-free asset market for trading periods $t = 1, \dots, T$. The economy applies Euro (€) as the currency. The risky asset considered in the market has no dividend. Moreover, the risky asset is traded in integer units, i.e. traders can trade 19 shares of the risky asset but not 19.81 shares. The risk-free asset is divisible in any trading quantity with the trading price normalized to 1. It has a constant interest factor $R = 1 + r$ with r denoting the nominal interest rate. N traders participate in the economy to trade in both markets. The economy considers no transaction cost and no short sale constraint for traders.

The risky asset market is a Xetra auction market that holds one Xetra auction for each trading period to determine the market price and the trading volume by the market mechanism of XAMM. Specifically, XAMM opens the market at the call phase, disseminating the real-time information of the central order book. Upon observing the real-time trading data from the market, traders perform their investment decisions and submit orders. XAMM then collects the submitted orders to the central order book and simultaneously updates the real-time trading data. The call phase stops randomly after a fixed time span and the price determination phase follows to determine Xetra auction price and the final transaction volumes. After that, XAMM cancels the unexecuted part of the orders and conducts the settlement to complete the payment for each transaction. After trading in Xetra auction market, traders interact with the risk-free asset market and trade for the risk-free asset.

We regard the risk-free asset market as the environment of Xetra auction market. Given the information flows from the risk-free asset market, the interactions among traders and XAMM provide the functionality of Xetra auction market, i.e. determining the trading price and reallocating the risky asset among traders. The first step for developing the agent-based model of XAMS is to specify constructive aspects of the system as follows.

Example 1 (**Constructive aspects of XAMS**).

> **I.** XAMS considers economic entities which operate in the market, i.e. N traders, XAMM, the numeraire employed in the market, and the risky asset traded in the market. The environment of XAMS is the risk-free asset market.
>
> **II.** XAMS connects to the risk-free asset market to request the information of the interest factor R as well as to transmit trader's trading request on the risk-free asset. The risk-free asset market connects to XAMS to inform traders the interest factor R as well as their realized trading quantities of the risk-free asset. These information flows represent the interrelation between XAMS and its environment.
>
> **III.** XAMS is regarded as a dynamical system. We apply the concept of the system clock to represent the time horizon considered in the model. Thus, elements of XAMS are: N traders, XAMM, the risky asset, the numeraire, and the system clock.[1]
>
> **IV.** Consider a decentralized market. Traders connect with XAMM in order to perform the trading behavior, whereas there is no direct connection among traders. Traders and XAMM connect to the risky asset, the numeraire, and the system clock to have access to the associated information. Denote Xetra auction market center in the agent-based model as a composite of XAMM, the numeraire, the risky asset, and the system clock. Then the structure of XAMS follows the type of the star network, see Figure 1. Xetra auction market center as the central node in the star network connects with all other nodes of traders in the agent-based model. □

[1] One can exclude listing the numeraire and the system clock as the elements of XAMS, by assuming these two elements are automatically attached in the agent-based model.

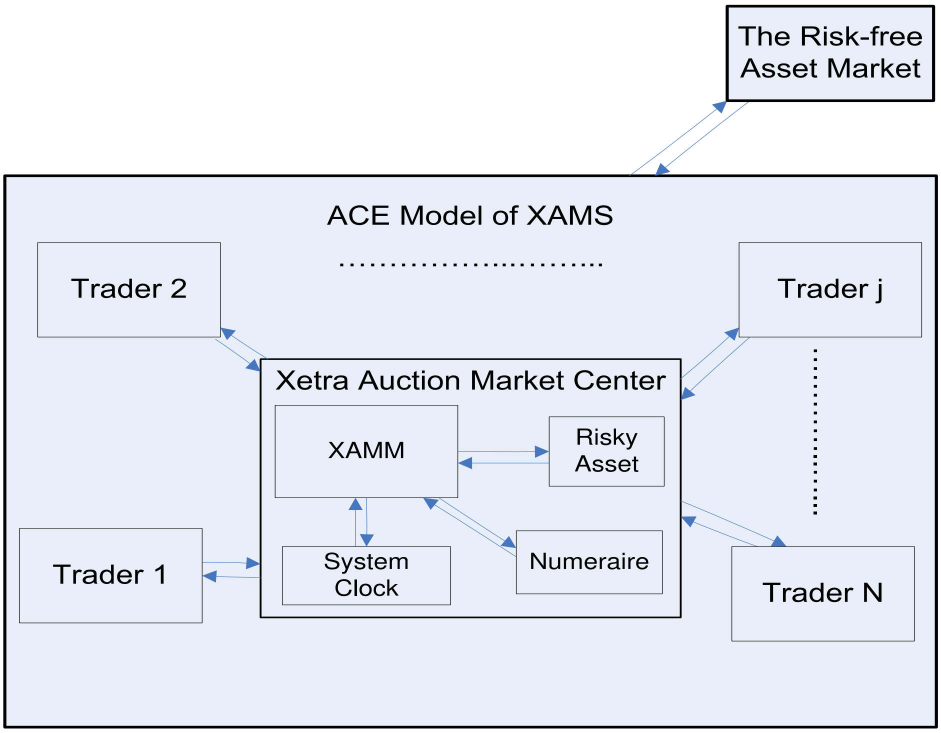

Figure 1: Network diagram for the agent-based model of XAMS.

Classify agents in XAMS as active economic agents of N traders and XAMM with passive economic agents of the risky asset, the numeraire, and the system clock. The second step for standardized agent-based modeling is to construct these agents respectively.

We consider three types of traders in XAMS. The first type is the price setter who assumes that, with the knowledge on XAMM and the real-time trading data, it could manipulate the current trading price as well as its transaction volume by its trading behavior. The second type is the price taker who believes that its trading behavior has no impact on the market. The last type is the noise trader who is assumed to act randomly in the market. Assume trader 1 in the agent-based model as the price setter, trader $j \in \{2, \ldots, N-1\}$ as price takers, and trader N as the noise trader. For simplicity, further assume that each trader submits at most one order in each trading period with price setter and noise trader submitting market order and price takers submitting limit order. We follow Module of Active Economic Agent (MAEA) proposed in Li (2014) as the blueprint for modeling active economic agent, with emphasis on agent's information acquirement, forecasting, objective, and action transmission. We construct trader 1 in Example 2, trader $j \in \{2, \ldots, N-1\}$ in Example 3, and trader N in Example 4 respectively.

Example 2 (Trader 1). Trader 1 connects with Xetra auction market center and the risk-free asset market. At the beginning of the trading period t, trader 1 is with the initial endowment $(y_0^{(1)}[t], Z_0^{(1)}[t])$ where $y_0^{(1)}[t]$ is the initial holding of the risk-free asset and $Z_0^{(1)}[t]$ is the initial holding of the risky asset. Trader 1 obtains the information of the interest factor R from the risk-free asset market and the real-time data set $\mathcal{I}_0[t]$ of the central order book from XAMM.

Then trader 1 computes its forecast on the expected mean value $q^{e(1)}[t]$ of the risky asset price for the next trading period $t+1$ and its associated variance $V^{e(1)}[t]$. As trader 1 can manipulate the current trading price as well as its transaction volume by submitting its market order, it performs its subjective forecast $P_X^{e(1)}[t](Q_m)$ on the current Xetra auction price and $Z_X^{e(1)}[t](Q_m)$ on the final transaction volume for period t which are functions with the control variable Q_m of the quoted trading quantity in its market order. The values of $P_X^{e(1)}[t](Q_m)$ and $Z_X^{e(1)}[t](Q_m)$ are the trading price and the final transaction volume that XAMM would realize by perceiving the trader's market order with quoted trading quantity Q_m would be added to the current central order book in the market.

With its forecast of $\{P_X^{e(1)}[t](Q_m), Z_X^{e(1)}[t](Q_m), q^{e(1)}[t], V^{e(1)}[t]\}$, the trader has the budget constraint:

$$P_X^{e(1)}[t](Q_m) \cdot Z_X^{e(1)}[t](Q_m) + y^{e(1)}[t] = 0, \tag{1}$$

where $y^{e(1)}[t]$ is the trader's expected trading quantity of the risk-free asset in period t. The trader expects the portfolio holding after trading in period t as $(y_0^{(1)}[t] + y^{e(1)}[t], Z_0^{(1)}[t] + Z_X^{e(1)}[t](Q_m))$. With the budget constraint (1), the trader considers the mean value $mean^{(1)}[t]$ of its future wealth at the end of the trading period t as:

$$\begin{aligned} mean^{(1)}[t] &= \{q^{e(1)}[t] - R \cdot P_X^{e(1)}[t](Q_m)\} \cdot Z_X^{e(1)}[t](Q_m) \\ &\quad + q^{e(1)}[t] \cdot Z_0^{(1)}[t] + R y_0^{(1)}[t]. \end{aligned} \tag{2}$$

The associated variance $var^{(1)}[t]$ is as:

$$var^{(1)}[t] = \{Z_0^{(1)}[t] + Z_X^{e(1)}[t](Q_m)\}^2 \cdot V^{e(1)}[t]. \tag{3}$$

We assume that trader 1 takes the linear mean-variance preference. The trader presents its objective as the portfolio selection problem:

$$\begin{aligned} & \max_{Q_m \in \mathbb{Z}} \quad mean^{(1)}[t] - \frac{\alpha_1}{2} var^{(1)}[t] && (4) \\ \Leftrightarrow \; & \max_{Q_m \in \mathbb{Z}} \quad \{q^{e(1)}[t] - R \cdot P_X^{e(1)}[t](Q_m)\} \cdot Z_X^{e(1)}[t](Q_m) \\ & \quad + q^{e(1)}[t] \cdot Z_0^{(1)}[t] + R y_0^{(1)}[t] - \frac{\alpha_1}{2}\{Z_0^{(1)}[t] + Z_X^{e(1)}[t](Q_m)\}^2 \cdot V^{e(1)}[t], \end{aligned}$$

where α_1 is a constant measure of absolute risk aversion. Trader 1 solves this portfolio selection problem (4) and obtains the integer maximizer $Q_m^{(1)}[t]$ with $Q_m^{(1)}[t] > 0$ denoting the trading quantity on the demand side and $Q_m^{(1)}[t] < 0$ denoting the trading quantity on the supply side. Then the trader transmits its action by submitting the market order with the quoted trading quantity $Q_m^{(1)}[t]$ to XAMM.

Trader 1 realizes the trading price $P_X[t]$ and its transaction volume $Z_X^{(1)}[t]$ after XAMM determines the trading price and the trading volume in the price determination phase. Then the trader completes with XAMM the payment for its transaction.

After that, trader 1 obtains from the risk-free asset market the share $y^{(1)}[t] = -P_X[t] \cdot Z_X^{(1)}[t]$ of the risk-free asset. The portfolio holding that the trader acquires after trading in period t is $(y_0^{(1)}[t] + y^{(1)}[t], Z_0^{(1)}[t] + Z_X^{(1)}[t])$ and the trader's initial endowment of the next trading period $t+1$ is

$$\begin{cases} y_0^{(1)}[t+1] & = R(y_0^{(1)}[t] + y^{(1)}[t]), \\ Z_0^{(1)}[t+1] & = Z_0^{(1)}[t] + Z_X^{(1)}[t]. \end{cases} \tag{5}$$

The dynamics of trader 1, i.e. its decision making process, is illustrated in Figure 2. □

Example 3 (Trader $\mathbf{j = 2, \dots, N-1}$). Analogous to trader 1, trader j connects with Xetra auction market center and the risk-free asset market. At the beginning of the trading period t, trader j is with the initial endowment $(y_0^{(j)}[t], Z_0^{(j)}[t])$. The trader obtains the information of the interest factor R and the real-time order book data set $\mathcal{I}_0[t]$.

Trader j computes its forecast on the expected mean value $q^{e(j)}[t]$ of the risky asset price for the next trading period $t+1$ and its associated variance $V^{e(j)}[t]$. As the trader has to decide a limit price to quote in its limit order, the trader conducts its subjective forecast $P_X^{e(j)}[t]$ on the current Xetra auction price and regards its forecast as the limit price. The trader expects it will realize from the market the quoted trading quantity Q_l in its limit order.

With its forecast of $\{q^{e(j)}[t], V^{e(j)}[t], P_X^{e(j)}[t]\}$, the trader has the budget constraint:

$$P_X^{e(j)}[t] \cdot Q_l + y^{e(j)}[t] = 0, \tag{6}$$

where $y^{e(j)}[t]$ is the trader's expected trading quantity of the risk-free asset in period t. The trader expects the portfolio holding after trading in period t as $(y_0^{(j)}[t] + y^{e(j)}[t], Z_0^{(j)}[t] + Q_l)$. With the budget constraint (6), the trader considers the mean value $mean^{(j)}[t]$ of its future wealth at the end of the trading period t as:

$$mean^{(j)}[t] = \{q^{e(j)}[t] - R \cdot P_X^{e(j)}[t]\} \cdot Q_l + q^{e(j)}[t] \cdot Z_0^{(j)}[t] + R y_0^{(j)}[t]. \tag{7}$$

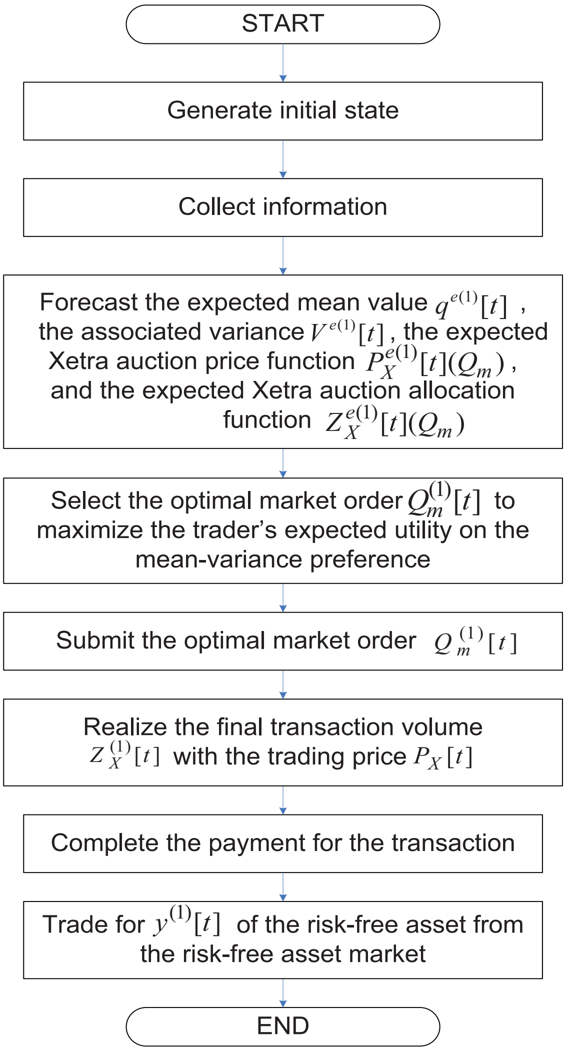

Figure 2: Decision making process of trader 1 in Example 2.

The associated variance $var^{(j)}[t]$ is as:

$$var^{(j)}[t] = \{Z_0^{(j)}[t] + Q_l\}^2 \cdot V^{e(j)}[t]. \tag{8}$$

Assume that trader j takes the linear mean-variance preference. The trader presents its objective as the portfolio selection problem:

$$\begin{aligned}
&\max_{Q_l \in \mathbb{Z}} \quad mean^{(j)}[t] - \frac{\alpha_j}{2}\, var^{(j)}[t] \qquad (9) \\
\Leftrightarrow\ &\max_{Q_l \in \mathbb{Z}} \quad \{q^{e(j)}[t] - R \cdot P_X^{e(j)}[t]\} \cdot Q_l \\
&\qquad + q^{e(j)}[t] \cdot Z_0^{(j)}[t] + Ry_0^{(j)}[t] - \frac{\alpha_j}{2}\{Z_0^{(j)}[t] + Q_l\}^2 \cdot V^{e(j)}[t],
\end{aligned}$$

where α_j is a constant measure of absolute risk aversion. Trader j solves this portfolio selection problem (9) and obtains the integer maximizer $Q_l^{(j)}[t]$. Then the trader transmits its action by submitting its limit order with the price-quantity pair $(P_X^{e(j)}[t], Q_l^{(j)}[t])$ to XAMM.

Trader j realizes the trading price $P_X[t]$ and its transaction volume $Z_X^{(j)}[t]$ after XAMM determines the trading price and the trading volume in the price determination phase. Then the trader completes with XAMM the payment for its transaction.

After that, trader j attains from the risk-free asset market the share of risk-free asset $y^{(j)}[t] = -P_X[t] \cdot Z_X^{(j)}[t]$. The portfolio holding that the trader acquires after trading in period t is $(y_0^{(j)}[t] + y^{(j)}[t], Z_0^{(j)}[t] + Z_X^{(j)}[t])$ and the trader's initial endowment of the next trading period $t+1$ is

$$\begin{cases} y_0^{(j)}[t+1] & = R(y_0^{(j)}[t] + y^{(j)}[t]), \\ Z_0^{(j)}[t+1] & = Z_0^{(j)}[t] + Z_X^{(j)}[t]. \end{cases} \tag{10}$$

The decision making process of trader j is illustrated in Figure 3. □

Example 4 (Trader N). Consider trader N is with the initial endowment of $(y_0^{(N)}[t], Z_0^{(N)}[t])$ at the beginning of the trading period t. Trader N obtains the information of the interest factor R from the risk-free asset market.

The trader conducts no forecasting, it simply selects $Q_m^{(N)}[t]$ randomly from the set Q_{range} of all possible trading quantities considered by the noise trader. Then the trader constructs its market order with the quoted trading quantity $Q_m^{(N)}[t]$ and submits the order to XAMM.

Trader N realizes the trading price $P_X[t]$ and its transaction volume $Z_X^{(N)}[t]$ after XAMM determines the trading price and the trading volume in the price determination phase. Then the trader completes with XAMM the payment for its transaction.

After that, trader N obtains from the risk-free asset market the shares of the risk-free asset $y^{(N)}[t] = -P_X[t] \cdot Z_X^{(N)}[t]$. The portfolio holding that the trader acquires after trading in period t is $(y_0^{(N)}[t] + y^{(N)}[t], Z_0^{(N)}[t] + Z_X^{(N)}[t])$ and the trader's initial endowment for the next trading period $t+1$ is

$$\begin{cases} y_0^{(N)}[t+1] & = R(y_0^{(N)}[t] + y^{(N)}[t]), \\ Z_0^{(N)}[t+1] & = Z_0^{(N)}[t] + Z_X^{(N)}[t]. \end{cases} \tag{11}$$

The decision making process of trader N is illustrated in Figure 4. □

XAMM is another type of active economic agent considered in the agent-based model. It has the objective of determining Xetra auction price and the final transaction volume according to the central order book.

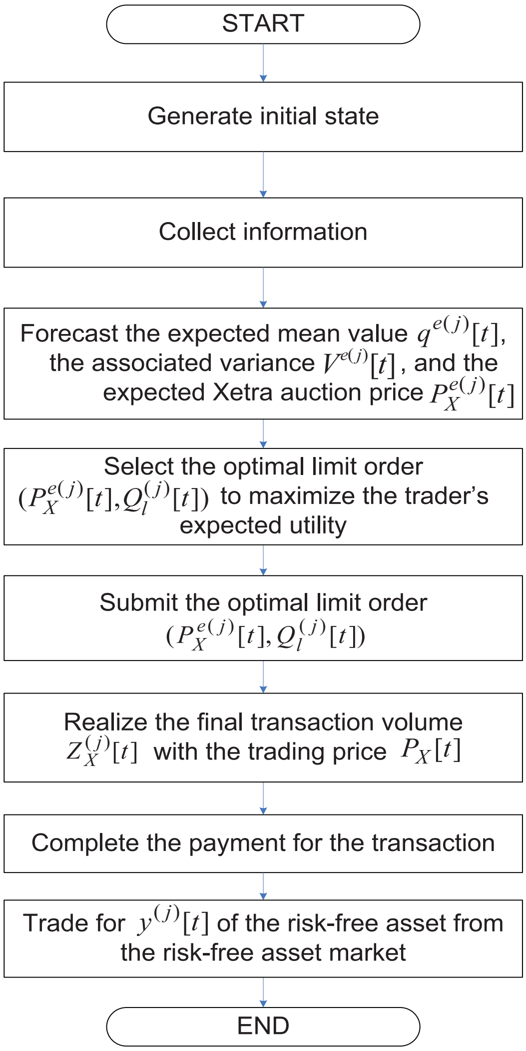

Figure 3: Decision making process of trader j in Example 3.

Example 5 (XAMM). At the beginning of the trading period t, XAMM has historical trading prices $P_X[i]$ for trading period $i \in \{-K_{\text{XAMM}} + 1, \dots, 0\}$ with the memory span $K_{\text{XAMM}} > 0$. It contains trading information $(\overline{\mathcal{I}_0[i]}$, $P_X[i]$, $Z_X[i])$ for trading period $i \in \{1, \dots, t-1\}$ where $\overline{\mathcal{I}_0[i]}$ is the order book data set at the end of the call phase, $P_X[i]$ is the Xetra auction price, and $Z_X[i] = \{Z_X^{(1)}[i], \dots, Z_X^{(N)}[i]\}$ is the collection of the final transaction volume $Z_X^{(j)}[i]$ for each trader $j \in \{1, \dots, N\}$. In summary, XAMM at the beginning of the trading period t is with the historical trading information set

$$\begin{aligned} Infor[t-1] \;=\; \{ \; & (\overline{\mathcal{I}_0[t-1]}, P_X[t-1], Z_X[t-1]), \dots, (\overline{\mathcal{I}_0[1]}, P_X[1], Z_X[1]), \\ & P_X[0], \dots, P_X[-K_{\text{XAMM}} + 1] \; \}. \end{aligned} \tag{12}$$

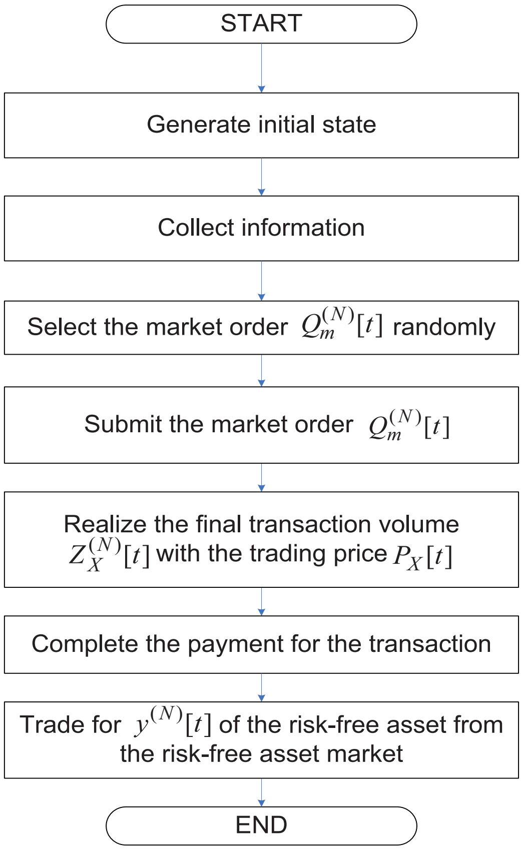

Figure 4: The decision making process of trader N in Example 4.

XAMM starts the trading period with the call phase. During the call phase, XAMM collects and stores order specifications $\{Q_m^{(1)}[t], \ldots, (P_X^{e(j)}[t], Q_l^{(j)}[t]), \ldots, Q_m^{(N)}[t]\}$ submitted by traders. It simultaneously disseminates the real-time order book data set $\mathcal{I}_0[t]$.

Forecast is not required in XAMM. The objective of XAMM is to determine in the price determination phase the Xetra auction price $P_X[t]$ and the final transaction volumes $Z_X[t] = \{Z_X^{(1)}[t], \ldots, Z_X^{(N)}[t]\}$ by applying Xetra auction trading rules stated in Deutsche Börse Group (2013), see the formulation in Appendix A. After determining Xetra auction price and the trading volume, XAMM cancels the unexecuted part of order specifications and conducts the settlement process to complete the payment for each transaction. Then XAMM closes the market until the next trading period $t+1$. The dynamics of XAMM are depicted in Figure 5. □

The numeraire, the risky asset, and the system clock are passive economic

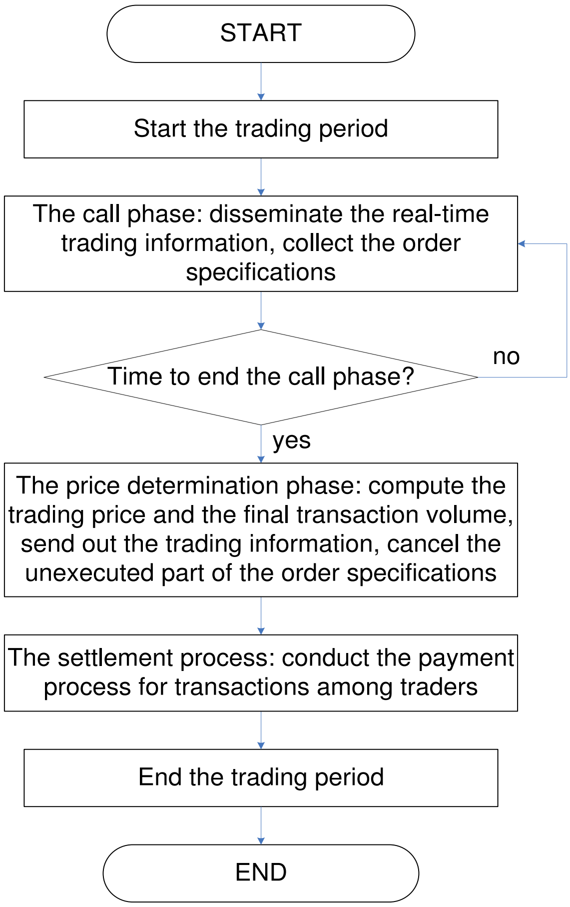

Figure 5: Dynamics of XAMM.

agents considered in the agent-based model. They act as information providers to provide on request the information about the currency employed in the market, the risky asset traded in the auction market, and the time considered in the model. As the environment of the agent-based model, the risk-free asset market provides the trading on the risk-free asset.

We consider dynamics of XAMS with T trading periods. XAMM starts the call phase at the beginning of the trading period. It disseminates to traders the real-time trading information of the central order book and simultaneously collects order specifications submitted by traders. Assume traders submit their order specifications in a random sequence during the call phase. To simplify our analysis, we further assume that price takers submit limit orders prior to the price setter and noise trader submit their market orders. The call phase stops randomly after a fixed time span and is followed by the price determination phase. XAMM determines in the price determination phase the Xetra auction price and the final transaction volume. Then it cancels the unexecuted part of the orders and conducts the settlement process to complete the payment for each transaction. After trading in the Xetra auction market, traders obtain the risk-free asset holdings via the interaction with the risk-free asset market. XAMS iterates to the next trading period until it reaches the last trading period T. The diagram of agent interaction in Figure 6 explicitly illustrates the sequence of activities among agents in the system dynamics.

3 Market Experiment

We implement the agent-based model of XAMS by applying the computer language Groovy/Java with the database back-end of Microsoft Access. Then we construct the market experiment and conduct the computer simulation of XAMS. The focus of the market experiment is on the generated dynamics of Xetra auction price.

3.1 Experimental Setup

We setup the simulation profile for the Xetra auction market experiment by initializing the parameters and by specifying the forecasting methods employed by agents.

Model's Parameters to Initialize

- $T = 250 \ldots$ time horizon. The time horizon approximates the time span of one year when considering one auction for each trading day and around 250 trading days for Frankfurt Stock Exchange in one year.
- $N = 22 \ldots$ number of traders. Three types of traders are considered in the model with 1 price setter, 20 price takers, and 1 noise trader.
- $r \ldots$ the interest rate of the risk-free asset. The interest rate is assumed to be constant in each profile. According to the Eurostat[2], the 3-months interest rate in European Union (27 countries) for the period of October 2008 to September 2009 is in the range of $[1.04\%, 5.52\%]$. We choose r randomly from this range.

[2]See http://epp.eurostat.ec.europa.eu.

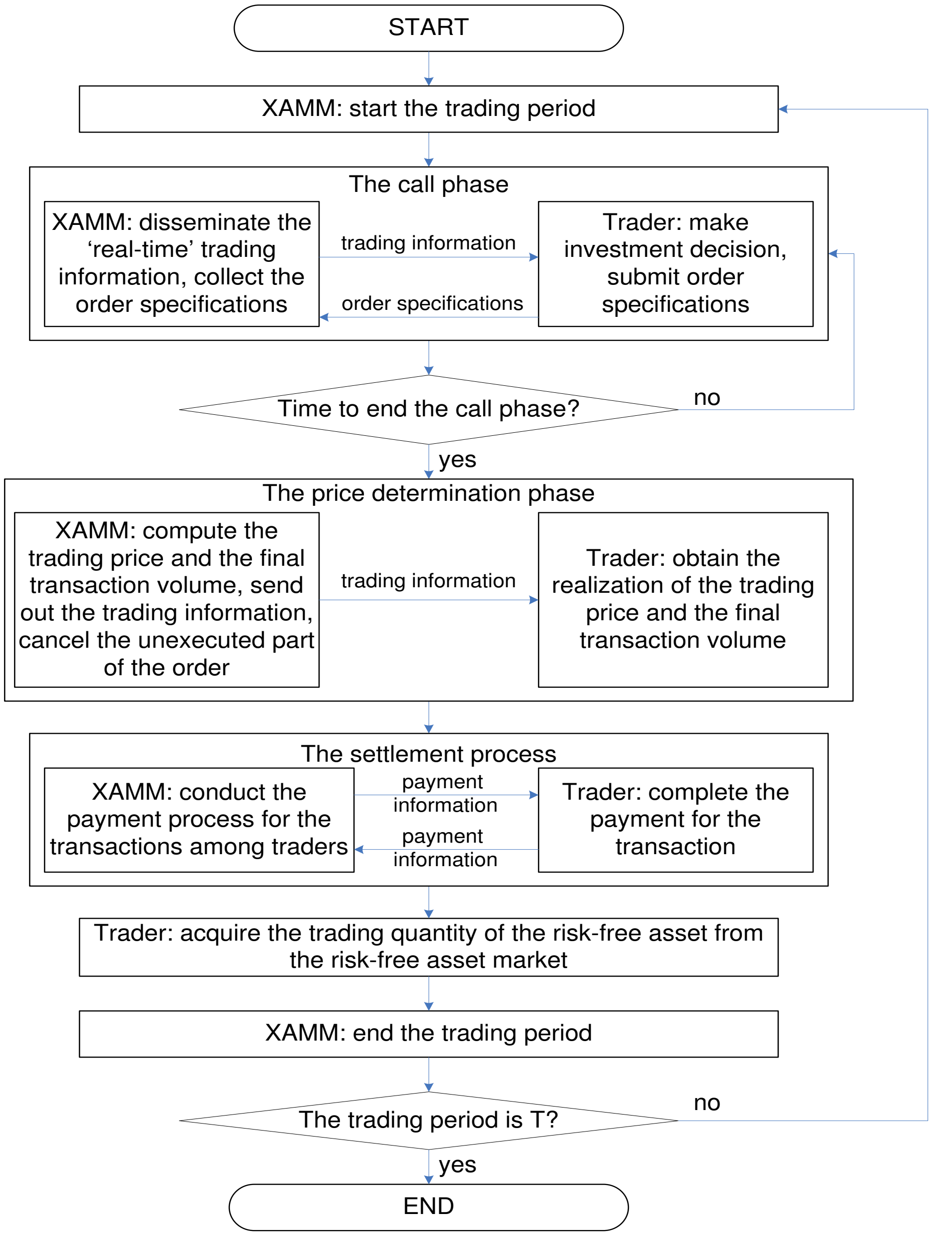

Figure 6: Diagram of agent interaction for the agent-based model of XAMS.

XAMM's Parameters to Initialize

- $\{P_X[0], \ldots, P_X[-K_{\text{XAMM}}+1]\}\ldots$ the historical trading prices. We consider $P_X[0], \ldots, P_X[-K_{\text{XAMM}}+1]$ as historical auction prices of the stock "Deutsche Börse AG" listed in Xetra for the period from August 27, 2009 to November 04, 2009, with the memory span $K_{\text{XAMM}} = 100$.[3]

- $\text{range}_\text{p} = 10\%\ldots$ the percentage of the price range. The Xetra platform requires transactions executed under certain price range from the last traded price P_{ref}. While it does not publicly provide the information of the percentage of the price range, another electronic trading platform Euronext requires the percentage of $\pm 10\%$. We employ the setting in Euronext and choose $\text{range}_\text{p} = 10\%$. Thus, XAMM considers Xetra auction price in the range of $[P_{\text{ref}}(1-10\%), P_{\text{ref}}(1+10\%)]$.

Trader's Parameters to Initialize

- $y_0^{(j)}[1]\ldots$ trader j's initial risk-free asset holding at the beginning of the trading period 1. We take $y_0^{(j)}[1]$ as a random positive number.
- $Z_0^{(j)}[1]\ldots$ trader j's initial risky asset holding at the beginning of the trading period 1. We take $Z_0^{(j)}[1]$ as a random integer number. To simplify the analysis, we assume that the aggregated volume in the market is constant with $\sum_{j=1}^{22} Z_0^{(j)}[1] = 1000$.
- $\alpha^{(j)}\ldots$ the measure of absolute risk aversion in the trader's utility function with the linear mean-variance preference. $\alpha^{(j)}$ is assumed to be constant in each profile and is selected randomly from the range of $(0, 2]$.
- $Q_{\text{range}}\ldots$ the set of the trading behavior for the noise trader $j = 22$ depicted in Example 4.

  We assume that the noise trader randomly chooses for each trading period the trading behavior from the set of $Q_{\text{range}} = \{$ selling 1 unit, buying 1 unit $\}$. We keep the noise trader's random choice of trading behavior for each period the same in the benchmark market experiment as in the Xetra auction market experiment.

Trader's Forecasting Methods to Initialize

1. Forecasting Methods in Common

   For each period t, trader $j = 1$ depicted in Example 2 and price takers $j \in \{2, \ldots, 21\}$ depicted in Example 3 compute the expected mean value $q^{e(j)}[t]$ of the risky asset price for the next trading period $t+1$ and its associated variance $V^{e(j)}[t]$. We assume two types of forecasting: the fundamentalist and the chartist. The forecasting type remains unchanged in each profile after trader j randomly chooses the forecasting type with equal probability.

[3] This historical data of the stock trading price is provided online by Deutsche Börse, see http://deutsche-boerse.com.

(a) The Fundamentalist

- $q^{e(j)}[t] \ldots$ The fundamentalist believes the mean value and the associated variance of the market price remain unchanged. It computes $q^{e(j)}[t]$ as the mean value of the historical trading prices $\{P_X[0], \ldots, P_X[-K_{\text{XAMM}}+1]\}$ with

$$q^{e(j)}[t] = \frac{1}{K_{\text{XAMM}}} \sum_{n=1}^{K_{\text{XAMM}}} P_X[1-n].$$

- $V^{e(j)}[t] \ldots$ The fundamentalist computes $V^{e(j)}[t]$ as the associated variance of the historical trading prices $\{P_X[0], \ldots, P_X[-K_{\text{XAMM}}+1]\}$ with

$$V^{e(j)}[t] = \frac{1}{K_{\text{XAMM}}-1} \sum_{n=1}^{K_{\text{XAMM}}} (P_X[1-n] - q^{e(j)}[t])^2.$$

The fundamentalist has constant forecast of $q^{e(j)}[t]$ and $V^{e(j)}[t]$ for each trading period.

(b) The Chartist

- $q^{e(j)}[t] \ldots$ The chartist expects the trend of the price movement based on the historical price movement. There are two types of chartists in our simulation: the trend follower and the contrarian. The trend follower expects the trading price will increase (decrease) given the trading price increased (decreased) in the last trading period while the contrarian expects the opposite. Let the indicator $\text{id}_\text{c} = 1$ for trend follower and $\text{id}_\text{c} = -1$ for contrarian. With historical trading prices $P_X[t-1]$ and $P_X[t-2]$, the forecasting of the chartist at period t is

$$q^{e(j)}[t] = \begin{cases} P_X[t-1](1 + \text{id}_\text{c} \cdot \omega^{(j)}) & \text{if } P_X[t-1] > P_X[t-2], \\ P_X[t-1] & \text{if } P_X[t-1] = P_X[t-2], \\ P_X[t-1](1 - \text{id}_\text{c} \cdot \omega^{(j)}) & \text{if } P_X[t-1] < P_X[t-2]; \end{cases}$$

  where $\omega^{(j)}$ measures how aggressive of the price movement that the trader expects.
  id_c and $\omega^{(j)}$ are assumed to be constant after id_c is chosen randomly from $\{-1, 1\}$ with equal probability and $\omega^{(j)}$ is chosen randomly from the range of $(0, \text{range}_\text{p}]$.
- $V^{e(j)}[t] \ldots$ It is assumed that the chartist keeps $V^{e(j)}[t]$ constant in the profile after it is chosen randomly from the range $(0, 5]$.

2. Forecasting Method for Price Setter $j = 1$ depicted in Example 2

- $P_X^{e(1)}[t]\ldots$ the forecast on Xetra auction price in the current trading period t. By applying formulation (16) in Appendix B, the price setter computes the forecast $P_X^{e(1)}[t](Q_m^{(1)}[t])$ that is a function of the quoted trading quantity $Q_m^{(1)}[t]$ in its market order.
- $Z_X^{e(1)}[t]\ldots$ the forecast on the trader's final transaction volume in the current trading period t. By applying formulation (17) in Appendix B, the price setter computes $Z_X^{e(1)}[t](Q_m^{(1)}[t])$ that is a function of the quoted trading quantity $Q_m^{(1)}[t]$ in its market order.

3. Forecasting Method for Price Taker $j = 2, \ldots, 21$ depicted in Example 3

- $P_X^{e(j)}[t]\ldots$ the forecast on Xetra auction price in the current trading period t. It is assumed that $P_X^{e(j)}[t]$ is randomly chosen from the price range $[P_{\text{ref}}(1-10\%), P_{\text{ref}}(1+10\%)]$ stipulated in XAMM.

3.2 Experimental Procedure

To investigate the market dynamics and the impact of the price setter in Xetra auction market, we construct along with the Xetra auction market experiment a benchmark market experiment. The benchmark market experiment has the same setup as the Xetra auction market experiment except for the price setter $j = 1$ depicted in Example 2. The benchmark market experiment replaces the price setter $j = 1$ with the benchmark trader which follows the same setup as depicted in Example 2 except for adopting the forecast $Z_X^{e(1)}[t](Q_m) = Q_m$ and $P_X^{e(1)}[t](Q_m) = P_{\text{ref}}$ where P_{ref} is the last traded price in Xetra auction market. Thus, the benchmark trader considers its objective as the portfolio selection problem:

$$\begin{aligned}\max_{Q_m \in \mathbb{Z}} \quad & \{q^{e(1)}[t] - R \cdot P_{\text{ref}}\} \cdot Q_m + q^{e(1)}[t] \cdot Z_0^{(1)}[t] + R y_0^{(1)}[t] \\ & - \frac{\alpha_1}{2}\{Z_0^{(1)}[t] + Q_m\}^2 \cdot V^{e(1)}[t], \end{aligned} \tag{13}$$

where α_1 is a constant measure of absolute risk aversion. As observed in (13), the trader considers in its portfolio selection problem that the current trading price and the trader's final trading volume are independent of Q_m. The benchmark trader is thus regressed to a price taker who has no capability of price manipulation.

We consider 50 rounds of market experiments. It starts with initiating 50 simulation profiles for each round of the market experiment with the index $s \in \{1, \ldots, 50\}$. Then for each profile s, we conduct the benchmark market experiment and the Xetra auction market experiment with 250 trading periods of simulations.

3.3 Experimental Results

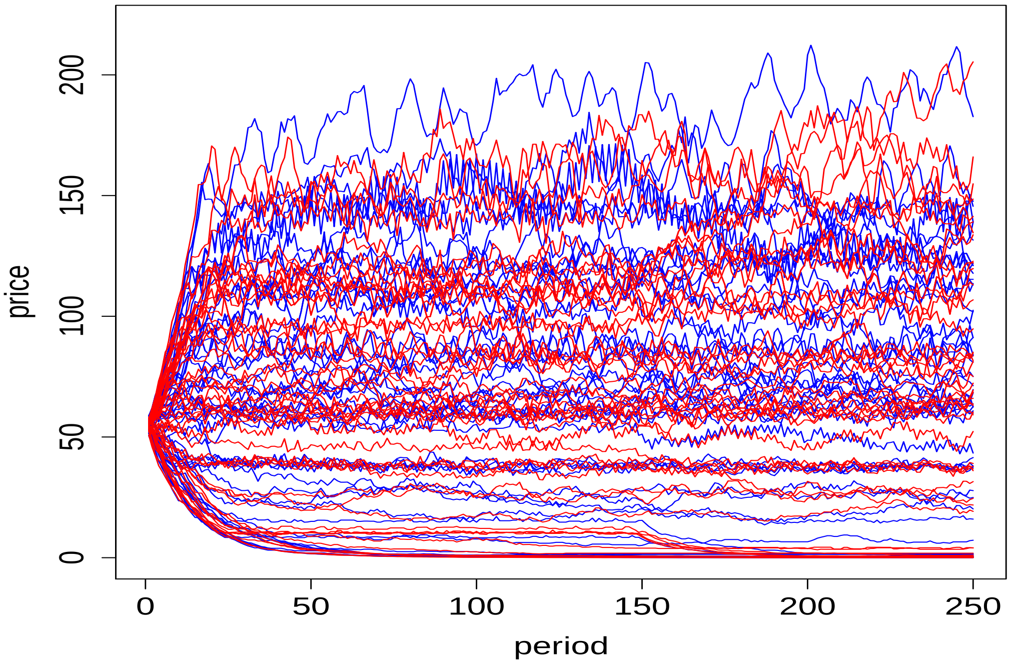

Figure 7: The price dynamics in 50 profiles.

Price Dynamics. We consider the price dynamics for 250 trading periods generated in the market experiment. Figure 7 illustrates the price dynamics generated by all 50 simulation profiles. The blue color in this figure as well as in the following figures is for the benchmark market experiment and the red color is for the Xetra auction market experiment.

Figure 7 demonstrates the divergence of the price dynamics in the market experiment. Consider the end-of-period trading price as the market price in the last trading period $t = 250$. It ranges $[0,\ 182.67]$ for all 50 benchmark market experiments and $[0,\ 205.39]$ for all 50 Xetra auction market experiments in the simulation. The histogram of the end-of-period trading price depicted in Figure 8 illustrates the divergence emerges in both market experiments.

Property of Non-equilibrium Trading Price. We employ the percentage of the non-equilibrium trading price $percent_{non}^{(s)}$ to investigate the non-equilibrium property of the trading price for each profile $s \in \{1, \dots, 50\}$ with

$$percent_{non}^{(s)} = \frac{\sharp\{\text{trading periods with non market-clearing trading price}\}}{\sharp\{\text{trading periods with trading price}\}}.$$

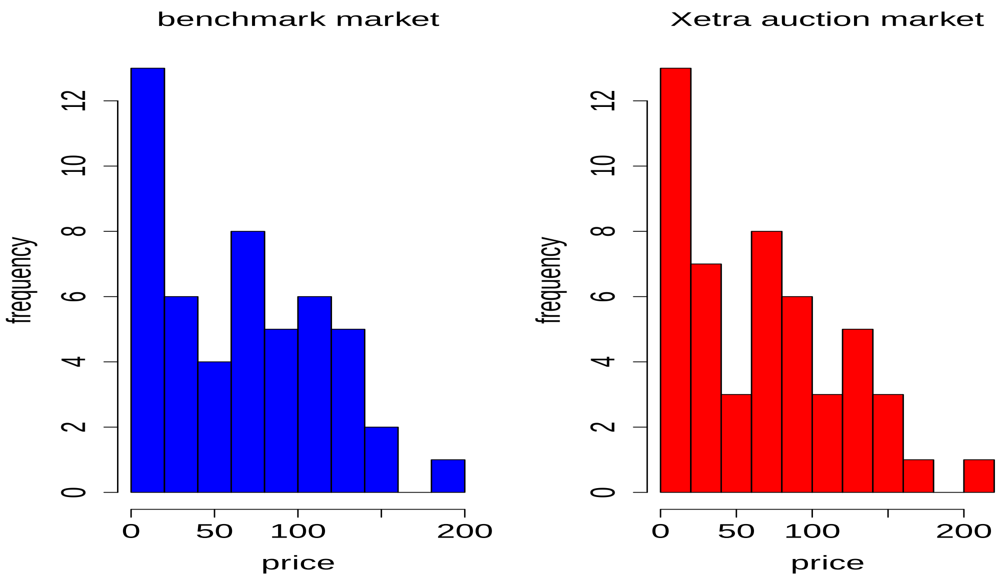

Figure 8: The histogram of the end-of-period trading price in the market experiments.

Figure 9 shows the percentage for the benchmark market experiment and for the Xetra auction market experiment in each profile. The lowest percentage of non-equilibrium trading price is 41.82% and the highest percentage is 96.58% in our simulation results, which implies that Xetra auction price generated in the market experiments is generically non-equilibrium. This property can be easily observed in Figure 9 where the blue line and the red line are far above the x-axis.

We conduct statistical test to investigate the impact of the price setter on the percentage of the non-equilibrium trading price. We specifically test whether $percent_{non}^{(s)}$ associated with the benchmark market experiment is greater than that associated with the Xetra auction market experiment. We apply the non-parametric statistical test – the Wilcoxon signed ranks test – and verbally present the null hypothesis as: $percent_{non}^{(s)}$ in the benchmark market experiment is no greater than that in the Xetra auction market experiment. The test result has $\mathrm{p-value} = 0.0659 < 0.1$. Thus, we reject the null hypothesis with 90% level of confidence and accept that $percent_{non}^{(s)}$ in the benchmark market experiment is greater than that in the Xetra auction market experiment. Thus, the Xetra auction market experiment where the price setter participates has a higher percentage of market equilibrium than the benchmark market experiment. This implies that the introduction of the price setter increases the possibility of market equilibrium and thus increases the market efficiency in Xetra auction market.

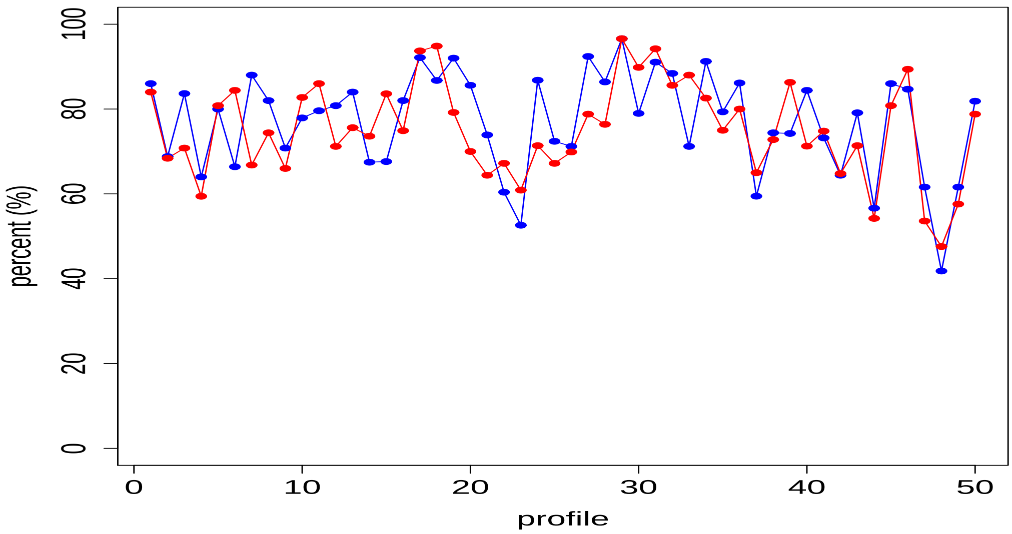

Figure 9: The percentage of non market-clearing price.

Intuitively, when Xetra auction market exists a surplus, i.e. the discrepancy between the aggregate demand and the aggregate supply, the price setter could submit a market order to accept the surplus without affecting the trading price in the market. When the price setter would submit a market order exceeding the surplus, the trading price would jump to an inferior position such that the trading price would fall down when the price setter would submit a market order in the sell side and vice versa. Thus the price setter has the incentive to meet but not outnumber the surplus in the market. When the price setter submits its market order to fully accept the surplus, Xetra auction market drives to equilibrium. The introduction of the price setter thus increases the market efficiency in Xetra auction market.

Price Volatility. We apply in this work the variance of the price dynamics $\{P_X^{(s)}[1], \ldots, P_X^{(s)}[250]\}$ for each profile s to measure price volatility in the market experiment. The variance of the price dynamics is formulated as:

$$Var^{(s)} = \frac{1}{249} \sum_{n=1}^{250} (P_X^{(s)}[n] - \overline{P_X^{(s)}})^2,$$

where $\overline{P_X^{(s)}}$ is the mean value of $\{P_X^{(s)}[1], \ldots, P_X^{(s)}[250]\}$. Figure 10 shows the variance for the benchmark market experiment and for the Xetra auction market experiment.

We use the Wilcoxon signed ranks test to investigate the impact of the price set-

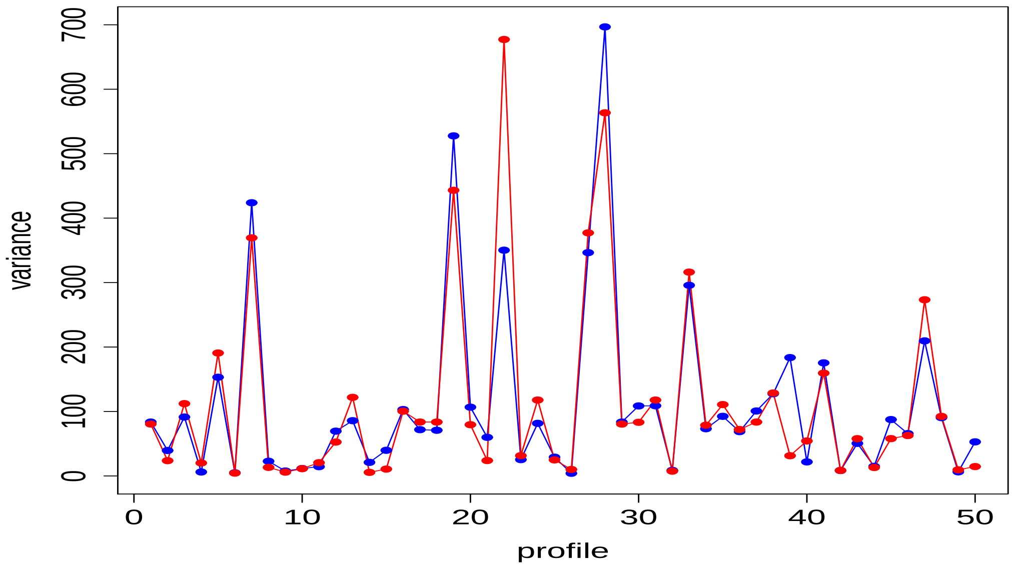

Figure 10: The variance of the trading price dynamics.

ter on the price volatility. The null hypothesis that we test is verbally presented as: the variance on the price dynamics for the benchmark market experiment has the same measure as that for the Xetra auction market experiment. The test result has $\mathrm{p-value} = 0.8545$. We can not reject the null hypothesis to accept that the variance on the price dynamics for the benchmark market experiment is significantly different from that for the Xetra auction market experiment. It seems that the introduction of the price setter does not significantly impact the price volatility in Xetra auction market.

Intuitively the price setter influences the price volatility of Xetra auction market in two different directions. The price setter exploits the market for a profit by its aggressive trading behavior, which would increase the price volatility in Xetra auction market. On the other hand, the introduction of the price setter increases the possibility of market equilibrium. The increase in market efficiency would allow traders to efficiently adjust their trading behavior to stabilize the market in equilibrium, which implies a decrease in the price volatility of the market. These two opposite impacts offset against each other so that the participation of the price setter would not significantly influence the price volatility in Xetra auction market.

4 Concluding Remarks

We have developed in this work the agent-based model of XAMS with the formulation of XAMM and of the price setter that represents HFT with the speed advantage to utilize the market transparency provided in Xetra auction market for profit exploitation. By implementing the agent-based model with the computer software system, we have conducted the computer simulation for market experiments. By investigating simulation results on market dynamics, we have discovered that the introduction of the price setter does not significantly influence price volatility in Xetra auction market. This finding implies that the role of algorithmic trading or HFT does not significantly generate negative impact on price volatility under the background of Xetra auction market with pre-trade transparency. Moreover, the investigation of simulation results on market dynamics has validated the property of non-equilibrium trading price in Xetra auction market. We have found by the market experiment that the introduction of the price setter improves market efficiency to some extent.

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A Xetra Auction Trading Rules

The Xetra auction trading rules are composed of the Xetra auction pricing rules and the allocation rules to determine the Xetra auction price and the trading volume respectively. Based on Deutsche Börse Group (2013), Figure 11 illustrates the Xetra auction pricing rules in which the subprocess Xetra-Auction-PDA($\mathcal{I}_0$) is depicted in Figure 12. The formulation of the Xetra auction allocation rules is illustrated in Figure 13.

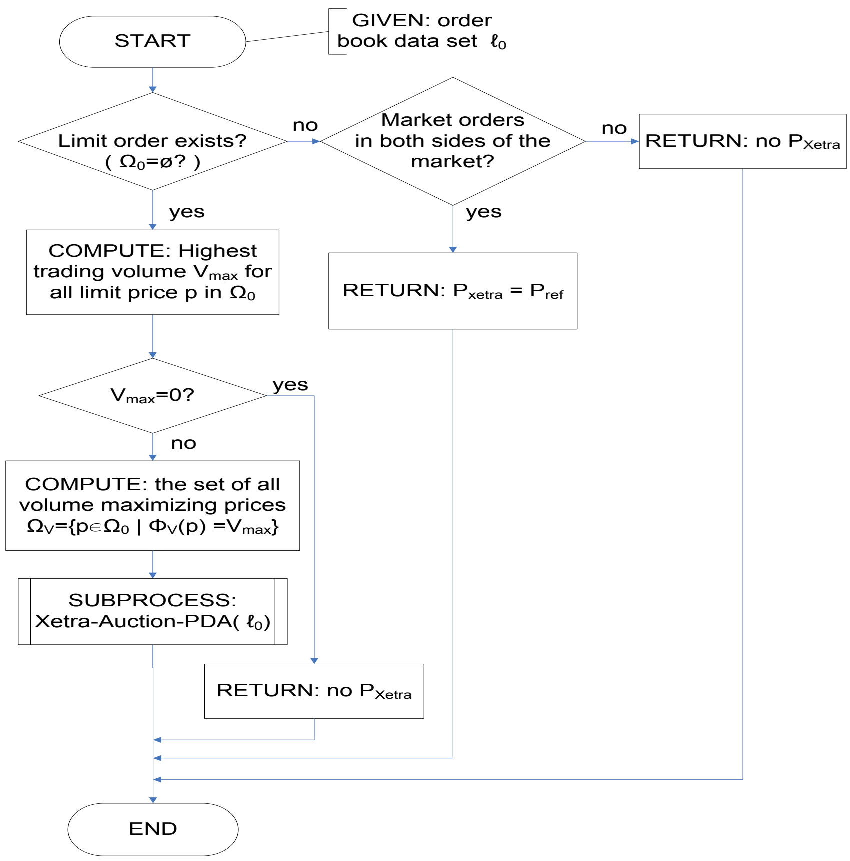

Figure 11: Flowchart of Xetra auction pricing rules.

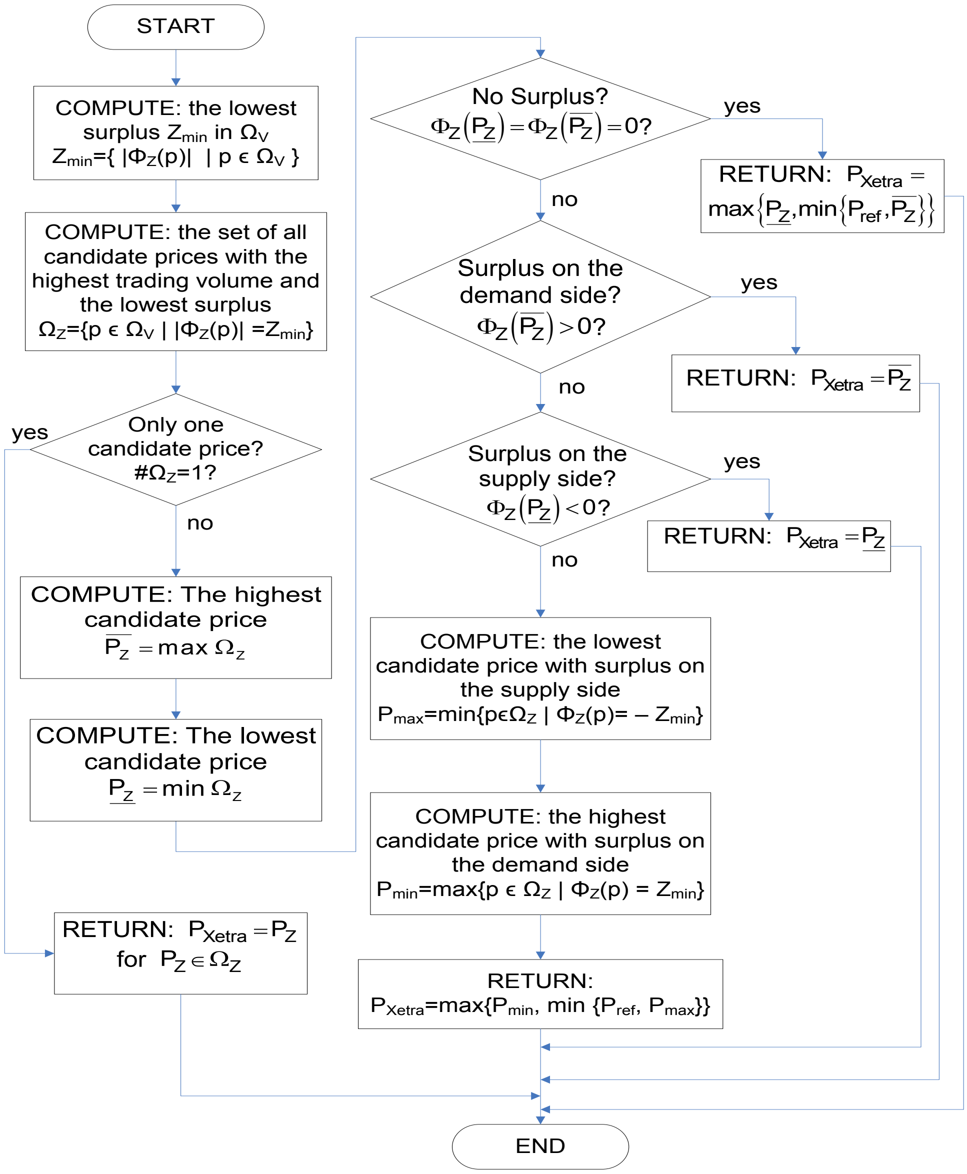

Figure 12: Flowchart of subprocess: Xetra-Auction-PDA($\mathfrak{I}_0$).

The function $QV(Ord)$ in Figure 13 returns the quoted trading quantity of the order Ord and the function $RV(Ord)$ denotes the realized trading volume for the order Ord.

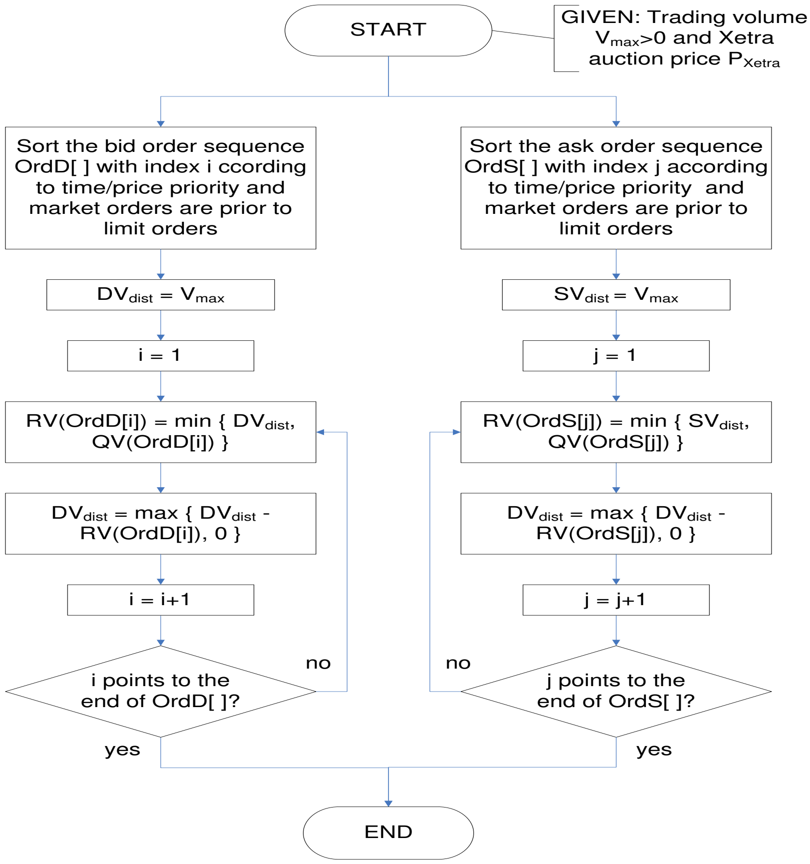

Figure 13: Flowchart of Xetra auction allocation rules.

B Subjective Forecast of Price Setter

We present the price setter's forecasting on the Xetra auction price and the final transaction volume. Introduce a new notation "$\lceil\!\rfloor$" to represent a closed half or an open half of an interval. For example, for any real numbers a and b with $a < b$, $\lceil\!\rfloor\, a, b\, \lceil\!\rfloor := [a, b]\ ,\ [a, b)\ ,\ (a, b]\ ,$ or (a, b). For trading period t, consider the central order book data set $\mathcal{I}_0[t]$ in Xetra auction contains a series of limit prices $P_1 \le P_2 \le \cdots \le P_{N_l}$ and the reference price P_{ref} that is the last trading price. The excess demand function for the order book $\mathcal{I}_0[t]$ is a step function with

$$\Phi_Z(p; \mathcal{I}_0[t]) = \sum_{n=0}^{N_l} \phi_n^Z 1_{A_n^Z}(p), \tag{14}$$

where ϕ_n^Z is a constant for $n = 0, \ldots, N_l$ and $\{A_0^Z, A_1^Z, \ldots, A_{N_l}^Z\}$ is a partition of $\mathbb{R}_+$ with $A_0^Z := [0, P_1\ \lceil\!\rfloor$; $A_n^Z := \lceil\!\rfloor\ P_n, P_{n+1}\ \lceil\!\rfloor$ for $n = 1, \ldots, N_l - 1$; and

$A^Z_{N_l} := [\, P_{N_l}, +\infty)$.

The price setter is to submit the market order $Q^{(1)}_m[t]$ with $Q^{(1)}_m[t] > 0$ denoting a market order on the demand side and $Q^{(1)}_m[t] < 0$ denoting that on the supply side. After the price setter submit its market order, the excess demand function updates to the form

$$\begin{aligned}\Phi'_Z(Q^{(1)}_m[t], p; \mathcal{I}_0[t]) &= \Phi_Z(p; \mathcal{I}_0[t]) + Q^{(1)}_m[t] \\ &= \sum_{n=0}^{N_l} \phi^Z_n 1_{A^Z_n}(p) + Q^{(1)}_m[t]. \end{aligned} \tag{15}$$

The price setter expects to be the last trader submitting order to the market. Then the price setter's forecast $P^e_X(Q^{(1)}_m[t])$ on the upcoming Xetra auction price is the step function:

$$P^e_X(Q^{(1)}_m[t]) = \begin{cases} P_1 & \text{when } Q^{(1)}_m[t] \in (-\infty, -\phi^Z_1); \\ P^*_n & \text{when } Q^{(1)}_m[t] = -\phi^Z_n, \quad n \in \{1, 2, \ldots, N_l - 1\}; \\ P_n & \text{when } Q^{(1)}_m[t] \in (-\phi^Z_{n-1}, -\phi^Z_n), \quad n \in \{2, 3, \ldots, N_l - 1\}; \\ P_{N_l} & \text{when } Q^{(1)}_m[t] \in (-\phi^Z_{N_l-1}, +\infty); \end{cases} \tag{16}$$

where for any $n \in \{1, 2, \cdots, N_l - 1\}$

$$P^*_n = \begin{cases} P_n & \text{CASE 1;} \\ P_{n+1} & \text{CASE 2;} \\ \max\{P_n, \min\{P_{\text{ref}}, P_{n+1}\}\} & \text{CASE 3;} \end{cases}$$

for **CASE 1:** either $A^Z_n = [P_n, P_{n+1})$ or $A^Z_n = (P_n, P_{n+1})$ with $|\Phi'_Z(-\phi^Z_n, P_n; \mathcal{I}_0[t])| < |\Phi'_Z(-\phi^Z_n, P_{n+1}; \mathcal{I}_0[t])|$;

for **CASE 2:** either $A^Z_n = (P_n, P_{n+1}]$ or $A^Z_n = (P_n, P_{n+1})$ with $|\Phi'_Z(-\phi^Z_n, P_n; \mathcal{I}_0[t])| > |\Phi'_Z(-\phi^Z_n, P_{n+1}; \mathcal{I}_0[t])|$;

for **CASE 3:** either $A^Z_n = [P_n, P_{n+1}]$ or $A^Z_n = (P_n, P_{n+1})$ with $|\Phi'_Z(-\phi^Z_n, P_n; \mathcal{I}_0[t])| = |\Phi'_Z(-\phi^Z_n, P_{n+1}; \mathcal{I}_0[t])|$.

Consider the notation $a^+ := \max\{0, a\}$ and $a^- := \min\{0, a\}$ for any $a \in \mathbb{R}$. The price setter's forecast $Z^e_X(Q^{(1)}_m[t])$ on the its final transaction volume is as:

$$Z^e_X(Q^{(1)}_m[t]) = \max\{(-\phi^Z_0)^-, \min\{Q^{(1)}_m[t], (-\phi^Z_{I+J})^+\}\}. \tag{17}$$